TASK: MAKING SENSE OF GRAPHS

ESSENTIAL QUESTIONS

- How can proportional relationships be described using the equation \( y = kx \)?
- How can proportional relationships be represented using rules, tables, and graphs?
- How can the graph of \( y = kx \) be interpreted for different contexts?
- How can algebraic expressions be used to model real-world situations?
- How can we solve simple algebraic equations, and how do we interpret the meaning of the solutions?

STANDARDS ADDRESSED

MCC.6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

MCC.6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

MCC.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form \( x + p = q \) and \( px = q \) for cases in which \( p, q \) and \( x \) are all nonnegative rational numbers.

MCC.6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

MCC.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations.

2. Reason abstractly and quantitatively. Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical equations and graphs.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments to compare the equation and graph.


5. Use appropriate tools strategically. Coordinate graph and tables.

6. Attend to precision. Students precisely place ordered pairs on the coordinate plane.

7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations.

8. Look for and express regularity in repeated reasoning. Students generalize that as “k” gets larger the line gets steeper.

MATERIALS
- graph paper
- colored pencils

TASK COMMENTS

In this task, students will create and analyze graphs to demonstrate their understanding of direct proportions. It is important for students to compare multiple graphs and describe the variation that occurs between the graphs and their corresponding equations. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.
TASK: SENSE OF GRAPHS

The graph below shows the amount of money required to buy gasoline if the cost per gallon is $2.00.

\[ y = 2x \]

a. What two quantities vary proportionally in this situation?

Solution

The price of gas varies proportionally with the number of gallons of gas purchased.

b. What is the value of the constant of proportionality? What does this value represent in the context of the problem? How is the constant of proportionality represented on the graph?

Solution

The constant of proportionality is 2. This value represents the price per gallon of gas.
c. Write an equation to represent this situation.
   
   **Solution**
   The constant of proportionality is 2 which means \( \frac{y}{x} = 2 \).
   Therefore, \( y = 2x \).

d. Suppose gas prices rose to $3.00 per gallon. How would the graph change? Explain your reasoning.
   
   **Solution**
   The graph would get steeper, because for every one increase in the number of gallons purchased, there is an increase of $3.00 in the cost instead of $2.00. Over the same amount of horizontal increase, there is a larger vertical increase, which would make the graph steeper.
e. Write an equation to represent the situation in part d.

Solution
The constant of proportionality is 3 which means $\frac{y}{x} = 3$.
Therefore, $y = 3x$.

Extension – change the gas price to $4.50 per gallon. Graph and explain the changes to the graph. Do this with several different prices for practice. Students could also come up with their own gas prices and explain how their graph looks. Questions to ponder – Are the graphs getting steeper? Where would a line be on the graph given a certain gas price?